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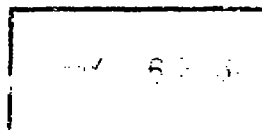
THE STABILITY OF THE COMPRESSION COVER OF BOX

BEAMS STIFFENED BY POSTS

By Paul Seide and Paul F. Barrett

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## TECHNICAL NOTE 2153

## THE STABILITY OF THE COMPRESSION COVER OF BOX

## BEAMS STIFFENED BY POSTS

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## SUMMARY

An investigation is made of the buckling of the compression cover of post-stiffened box beams subjected to end moments. Charts are presented for the determination of the minimum post axial stiffnesses and the corresponding compressive buckling loads required for the compression cover to buckle with nodes through the posts. Application of the charts to design and analysis and the limitations of their use are discussed.

If the flexural stiffness of the tension cover is equal to or less than that of the compression cover, buckle patterns with longitudinal nodes through the posts are not obtainable, no matter how stiff the posts are made or how closely they are spaced.

The weight of the required posts is found to be very low compared with the weight of the compression cover in an illustrative example, the ratio of the two being about  $2\frac{1}{2}$  percent.

## INTRODUCTION

Post-type construction has recently gained some support in the aircraft industry. This construction has been incorporated in at least one operational aircraft and is being considered for others. The basic elements are relatively thick tension and compression surfaces connected in the interior by posts or rods. These posts serve to raise the buckling load of the surfaces by restricting the buckle patterns that may form. In reference 1 and in the present paper, a compression surface supported by a rectangular array of rigid or effectively rigid posts is found to buckle as if continuously supported along either transverse or longitudinal lines through the posts.

If post-type construction is to reach an advanced stage of development, much research is needed. The large number of parameters involved,

even in the simplest idealization of the problem, however, makes a complete investigation difficult. The scope of the present investigation has therefore been very much limited.

The problem as conceived in the present paper is as follows: Two long, flat, rectangular plates of identical length and width, simply supported along their edges and of unequal thickness and material properties, are connected in the interior by identical axially elastic posts in rectangular array. The connection between the posts and plates is assumed to be one of simple support (of the ball-and-socket type, for instance) so that no resistance is offered to rotation of the plates. One plate is subjected to a longitudinal compressive load and the other, to an equal longitudinal tensile load. Both the longitudinal and transverse spacing of the posts are uniform; the two spacings, however, may differ. (See fig. 1.) The minimum axial stiffness of the posts required for the compression surface to buckle with nodes through the posts and the corresponding buckling loads are to be determined.

The structure thus outlined is an idealization of a long post-stiffened box beam subjected to end bending moments. The numerical results of a theoretical analysis of this problem are presented herein in both tabular and graphical form (tables I to III and figs. 2 to 4). The analysis itself, presented in the appendix, is of a more general nature and may be used to investigate other problems in the design of plate structures stiffened by posts.

## RESULTS AND DISCUSSION

The numerical results of the present investigation consist of values of the minimum post stiffnesses and the corresponding compressive buckling loads required for the compression surface of long post-stiffened box beams subjected to end bending moments to buckle with nodes through the posts. Because such buckle patterns do not transmit load to the posts, the posts are not compressed and are effectively rigid. Post axial stiffnesses greater than those presented herein will not serve to increase the buckling load of the structure. Combinations of the post axial-stiffness parameter and the compressive-buckling-load parameter, for different values of the ratio of the longitudinal distance between posts to the box-beam width and the ratio of the flexural stiffnesses of the tension and compression surfaces, are given in tables I to III and are presented graphically in figures 2 to 4 for one, two, and three rows of posts.

The symbols used in the presentation of the numerical results are the following which are also shown in figure 1:

$N_x b^2 / \pi^2 D_C$	compressive-buckling-load parameter (k)
$Fb^2 / \pi^2 D_C$	post axial-stiffness parameter
$L/b$	ratio of longitudinal distance between posts and box-beam width
$D_T/D_C$	ratio of flexural stiffnesses of tension and compression surfaces
$N_x$	critical load in tension and compression surfaces
$b$	box-beam width
$D$	plate flexural stiffness $\left( \frac{Et^3}{12(1 - \mu^2)} \right)$
$t$	plate thickness
$E$	Young's modulus of elasticity of plate material
$\mu$	Poisson's ratio of plate material
$F$	axial stiffness of posts, force per unit deflection
$L$	longitudinal distance between posts

The subscripts C and T refer to the compression and tension surfaces, respectively. The subscript cr refers to the transition from buckling with longitudinal nodes through the posts to buckling with transverse nodes through the posts.

The most significant qualitative result to be read from the figures (especially figs. 2(c), 3(c), and 4(c)) is that buckling of the compression surface with longitudinal nodes through the posts is never obtainable for box beams in use in aircraft construction (for which the stiffness of the tension surface is at most equal to that of the compression surface), even if the posts are infinitely stiff. Reference 1 shows that if the tension surface and the posts are rigid, buckling of the compression surface occurs with longitudinal nodes through the posts when the ratio of the distance between the posts and the box-beam width is less

than a certain value  $\left( \frac{L}{b} < \left( \frac{L}{b} \right)_{cr} \right)$  and with transverse nodes through the posts when this value is exceeded. These results are found to hold when the flexural stiffness of the tension surface is as little as about 3.5 times that of the compression surface (the exact value of  $D_T/D_C$  depends

on the number of rows of posts) and when the post axial stiffnesses are those given in figures 2(a), 3(a), and 4(a). However, if the flexural-stiffness ratio is less than this value of about 3.5, neither longitudinal nor transverse nodes through the posts can be obtained for some values of  $L/b$ , even if the post axial stiffness is made infinite. The spread of these values of  $L/b$  is indicated by figures 2(c), 3(c), and 4(c); the buckling loads in this region when the posts are rigid are shown by the dashed-line curves of figures 2(b), 3(b), and 4(b). As the flexural-stiffness ratio decreases, the region in which the compression cover buckles with longitudinal nodes through the posts also decreases until, when the ratio is about 1.5 or less, the longitudinal-node buckle pattern is not obtainable.

#### Limitations on Use of Curves in Analysis

The use of the curves of figures 2 to 4 for analysis is subject to many limitations. Inasmuch as the investigations of the present paper are limited in scope, the buckling loads of plates having post stiffnesses less than the stiffness required for nodes to pass through the posts cannot be found in most cases. Certain conclusions, however, may be drawn from the present results to aid in the determination of whether a given load will be carried by a given plate-post system:

(1) If, for given values of the ratio of the flexural stiffnesses of the tension and compression surfaces and the ratio of longitudinal distance between posts and box-beam width, the required buckling-load coefficient lies above the appropriate curve of figures 2(b), 3(b), or 4(b), the load cannot be carried because these curves indicate the maximum buckling loads of long plate-post structures.

(2) If the buckling-load coefficient lies on or below the solid or dash-dotted part of the appropriate curve and the post axial-stiffness parameter is greater than or equal to the value required for nodes to pass through the posts, as given by figures 2(a), 3(a), or 4(a), the load can be carried.

(3) If the buckling-load coefficient lies on the appropriate solid or dash-dotted line curves, and the post axial-stiffness parameter is less than the required value, the load cannot be carried. Further investigation is required if the buckling-load coefficient lies below these curves.

(4) If the buckling-load coefficient falls on the dashed part of the appropriate curve the load can definitely not be carried because an infinite post stiffness would be needed. Further investigation is necessary if the buckling-load coefficient is below these curves.

Several simplifying assumptions that also may affect the applicability of the present results to the design and analysis of actual post-stiffened

box beams have been made in the statement of the problem. The statement that the plates are flat and subjected to axial loading implicitly neglects the effects of bending of the plates and the resultant loads on the posts, due to the end moments. Possible buckling of the posts has not been considered but may be dealt with by making the moment of inertia of the post as large as possible. If hollow or built-up sections are used for this purpose, the thickness of the walls should be made large enough to prevent local buckling.

Because of the large number of computations involved, no investigations have been made to determine the adequacy of the present results in their application to plates of finite longitudinal length. It seems reasonable, however, in view of the results of references 3 to 5, to conclude that a finite plate having three posts or more in the longitudinal direction may be considered long, insofar as buckling with transverse nodes through the posts is concerned.

## APPLICATION TO ANALYSIS AND DESIGN

### Illustrative Design

As an illustration of the application of the various charts to a design problem, the posts are designed for a plate structure for which the following data are given:

Unit load, $N_x$ , pounds per inch . . . . .	15,000
Compression-plate thickness, $t_c$ equals tension-plate thickness, $t_T$ , inches . . . . .	0.375
Plate width, $b$ , inches . . . . .	28
Vertical distance between plates, $h$ , inches . . . . .	5.5
Modulus of elasticity of plate material, $E$ , psi . . . . .	$10.5 \times 10^6$
Poisson's ratio of plate material, $\mu$ . . . . .	0.3

It is desired that the buckle pattern be such that nodal lines pass through the posts. This specification is equivalent to that of designing the ribs of skin-stringer construction so that buckling takes place between ribs.

The required compressive-buckling-load coefficient of the structure is given by  $N_x b^2 / \pi^2 D_c$  where

$$\begin{aligned}
 D_C &= \frac{E_C t_C^3}{12(1 - \mu_C^2)} \\
 &= \frac{10.5 \times 10^6 \times (0.375)^3}{12(1 - 0.3^2)} \\
 &= 50,700 \text{ pound-inches}
 \end{aligned}$$

Then,

$$\begin{aligned}
 \frac{N_x b^2}{\pi^2 D_C} &= \frac{15,000 \times 28^2}{50,700 \pi^2} \\
 &= 23.5
 \end{aligned}$$

Inasmuch as the thickness and material properties of the tension plate are identical with those of the compression plate, the value of  $D_T/D_C$  for the problem is unity.

The maximum buckling-load coefficient obtainable with any number of rows of posts is given by  $\frac{N_x b^2}{\pi^2 D_C} = 4N^2$ , the buckling-load coefficient of a long plate of width  $b/N$ . From figure 2(b) it can be seen that the required buckling-load coefficient may not be obtained with one row of posts. The coefficient could be obtained by using two rows of posts but the required buckle pattern of nodes passing through the posts could not be obtained. At least three rows of posts must be used.

If three rows are used, the curves of figure 4(b) indicate that, for  $D_T/D_C$  equal to 1.00, a buckle pattern with transverse nodes through the posts is obtainable. With  $N_x b^2 / \pi^2 D_C$  equal to 23.5, the corresponding value of  $L/b$  is equal to 0.215. Hence, the maximum permissible longitudinal post spacing  $L$  is given by

$$\begin{aligned}
 L &= b \frac{L}{b} \\
 &= 28 \times 0.215 \\
 &= 6.0 \text{ inches}
 \end{aligned}$$

The minimum required post stiffness may now be obtained from figure 4(a).

With  $L/b$  equal to 0.215 and  $D_T/D_C$  equal to 1.00, the corresponding value of  $Fb^2/\pi^2D_C$  is equal to 300. The post stiffness is then

$$\begin{aligned} F &= \frac{\pi^2 D_C}{b^2} \frac{Fb^2}{\pi^2 D_C} \\ &= \frac{50,700\pi^2}{(28)^2} 300 \\ &= 191,000 \text{ pounds per inch} \end{aligned}$$

If the posts were of aluminum, the required area would be

$$\begin{aligned} A &= \frac{h}{E} F \\ &= \frac{5.5 \times 191,000}{10.5 \times 10^6} \\ &= 0.100 \text{ square inch} \end{aligned}$$

The diameter of a circular rod would be

$$\begin{aligned} d &= \sqrt{\frac{4}{\pi} \times 0.100} \\ &= 0.36 \text{ inch} \end{aligned}$$

The ratio of the weights of posts and compression cover is 0.026. This ratio is appreciably lower than that of the weights of ribs and compression surface in conventional skin-stringer construction, given in reference 2 as about 0.5 for minimum weight design, and represents a great saving in stiffening material. The question of whether such low-weight ratios are obtainable for actual box beams in which factors other than buckling of the compression cover must be considered in design cannot be completely answered until many designs utilizing post stiffening have been made.



## CONCLUSIONS

The minimum post-axial stiffness and the corresponding buckling loads required for the compression cover of a box beam stiffened by posts and subjected to end moments to buckle with nodes through the posts have been determined. Where buckling with nodes through the posts is not obtainable, even with rigid posts, the buckling loads of box beams with rigid posts are given.

If the flexural stiffness of the tension cover is equal to or less than that of the compression cover, buckle patterns with longitudinal nodes are not obtainable, no matter how stiff the posts are made or how closely they are spaced.

For the particular example worked in the present paper the weight of the required posts is very low compared with the weight of the compression cover, the ratio of the two being about  $2\frac{1}{2}$  percent. A complete evaluation of post-stiffening is not possible, however, until many more designs and tests have been made.

Langley Aeronautical Laboratory  
National Advisory Committee for Aeronautics  
Langley Air Force Base, Va., May 10, 1950

## APPENDIX

## DETAILS OF ANALYSIS AND COMPUTATIONS

## Symbols

b	width of plates
t	plate thickness
E	Young's modulus of elasticity of plate material
$\mu$	Poisson's ratio of plate material
D	plate flexural stiffness $\left( \frac{Et^3}{12(1 - \mu^2)} \right)$
R	plate flexural-stiffness ratio ( $D_T/D_C$ )
$N_x$	plate load per unit width
k	plate-load coefficient (buckling-load coefficient for compression plate) ( $N_x b^2 / \pi^2 D$ )
M	number of longitudinal bays
N	number of transverse bays
L	longitudinal distance between posts
$\beta$	aspect ratio of plate between adjacent lines of posts ( $L/b$ )
F	axial stiffness of posts, force per unit deflection
S	axial post-stiffness parameter ( $Fb^2 / \pi^2 D_C$ )
X,Y,Z	coordinate axes (see fig. 1)
x,y,z	distance along coordinate axes
w	plate deflection in Z-direction
$a_{mn}$	general coefficient in Fourier series for w
i,j,m,n,r,s	integers

c	integer defining location of post in x-direction ( $1 \leq c \leq (M - 1)$ )
d	integer defining location of post in y-direction ( $1 \leq d \leq (N - 1)$ )
q	integer defining number of buckles in x-direction ( $1 \leq q \leq (M - 1)$ )
p	integer defining number of buckles in y-direction ( $1 \leq p \leq (N - 1)$ )
U	potential energy
V	strain energy
W	work done by external loads

## Subscripts:

C	compression plate
T	tension plate
P	posts

## Derivation of Criteria for Stability of Plate

## Structures Stiffened by Posts

The following problem is more general than the one outlined in the introduction:

Two flat, rectangular plates of equal finite length and width, simply supported along their edges and of unequal thickness and material properties, are connected in the interior by identical axially elastic rods in rectangular array. The connection between the posts and plates is one of simple support, of the ball-and-socket type, for instance, so that no resistance is offered to rotation of the plate. The plates are subjected to unequal axial loads. Both the longitudinal and transverse spacing of the rods are uniform; the two spacings, however, may differ. (See fig. 1.) The combination of loads that will cause the structure to buckle is to be found.

The Rayleigh-Ritz energy method used to analyze this problem consists of the following procedure. Fourier series to represent the deflected shape of the plates are first assumed. The energy of the various components

of the structure and the work done by the external loads are then expressed in terms of the unknown Fourier coefficients, and the potential-energy expression of the structure is written. When the potential-energy expression is minimized with respect to each of the coefficients, a set of simultaneous equations is obtained which is manipulated in the manner of references 3 and 6 to yield finally the stability criteria for the structure.

The deflected shapes of the two buckled plates may be expressed generally by the double Fourier series

$$w_C = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mnC} \sin \frac{m\pi x}{ML} \sin \frac{n\pi y}{b} \quad (1)$$

$$w_T = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mnT} \sin \frac{m\pi x}{ML} \sin \frac{n\pi y}{b} \quad (2)$$

These expressions satisfy the condition of simple support on all four edges of each plate.

The energy stored in the buckled plates is then

$$\begin{aligned} V_C + V_T &= \frac{D_C}{2} \int_0^{ML} \int_0^b \left( \frac{\partial^2 w_C}{\partial x^2} + \frac{\partial^2 w_C}{\partial y^2} \right)^2 dx dy + \frac{D_T}{2} \int_0^{ML} \int_0^b \left( \frac{\partial^2 w_T}{\partial x^2} + \frac{\partial^2 w_T}{\partial y^2} \right)^2 dx dy \\ &= \frac{\pi^4 b}{8(ML)^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ m^2 + \left( \frac{ML}{b} \right)^2 n^2 \right]^2 \left( D_C a_{mnC}^2 + D_T a_{mnT}^2 \right) \end{aligned} \quad (3)$$

and that stored in the deformed posts is

$$\begin{aligned} V_P &= \sum_{c=1}^{M-1} \sum_{d=1}^{N-1} \frac{F}{2} (w_C - w_T)^2 \\ &= \frac{F}{2} \sum_{c=1}^{M-1} \sum_{d=1}^{N-1} \left[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (a_{mnC} - a_{mnT}) \sin \frac{n\pi c}{M} \sin \frac{n\pi d}{N} \right]^2 \end{aligned} \quad (4)$$

The work done by the loads in moving with the ends of the plates is

$$\begin{aligned}
 W_C + W_T &= \frac{N_{x_C}}{2} \int_0^{ML} \int_0^b \left( \frac{\partial w_C}{\partial x} \right)^2 dx dy - \frac{N_{x_T}}{2} \int_0^{ML} \int_0^b \left( \frac{\partial w_T}{\partial x} \right)^2 dx dy \\
 &= \frac{\pi^2 b}{8ML} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 (N_{x_C} a_{mn_C}^2 - N_{x_T} a_{mn_T}^2) \quad (5)
 \end{aligned}$$

The potential energy is

$$\begin{aligned}
 U &= V_C + V_T + V_P - W_C - W_T \\
 &= \frac{\pi^4 b}{8(ML)^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ m^2 + \left( \frac{ML}{b} \right)^2 n^2 \right]^2 (D_C a_{mn_C}^2 + D_T a_{mn_T}^2) + \\
 &\quad \frac{F}{2} \sum_{c=1}^{M-1} \sum_{d=1}^{N-1} \left[ \sum_m^{\infty} \sum_{n=1}^{\infty} (a_{mn_C} - a_{mn_T}) \sin \frac{m\pi c}{M} \sin \frac{n\pi d}{N} \right]^2 - \\
 &\quad \frac{\pi^2 b}{8ML} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 (N_{x_C} a_{mn_C}^2 - N_{x_T} a_{mn_T}^2) \quad (6)
 \end{aligned}$$

The buckling load is determined by the condition that the potential energy be a minimum; that is,

$$\frac{\partial U}{\partial a_{ij_C}} = \frac{\partial U}{\partial a_{ij_T}} = 0 \quad \begin{matrix} (i = 1, 2, \dots, \infty) \\ (j = 1, 2, \dots, \infty) \end{matrix}$$

or

$$\begin{aligned}
 &\left[ (i^2 + j^2 M^2 \beta^2)^2 - k_C i^2 M^2 \beta^2 \right] a_{ij_C} + \frac{4M^3 \beta^3 S}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (a_{mn_C} - \\
 &a_{mn_T}) \left( \sum_{c=1}^{M-1} \sin \frac{m\pi c}{M} \sin \frac{i\pi c}{M} \right) \left( \sum_{d=1}^{N-1} \sin \frac{n\pi d}{N} \sin \frac{j\pi d}{N} \right) = 0 \quad (7)
 \end{aligned}$$

and

$$\left[ (i^2 + j^2 M^2 \beta^2)^2 + k_T i^2 M^2 \beta^2 \right] a_{ijT} - \frac{4M^3 \beta^3 S}{\pi^2 R} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (a_{mnC} - a_{mnT}) \left( \sum_{c=1}^{M-1} \sin \frac{m\pi c}{M} \sin \frac{i\pi c}{M} \right) \left( \sum_{d=1}^{N-1} \sin \frac{n\pi d}{N} \sin \frac{j\pi d}{N} \right) = 0 \quad (8)$$

$$\begin{aligned} & (i = 1, 2, \dots, \infty) \\ & (j = 1, 2, \dots, \infty) \end{aligned}$$

Equations (7) and (8) may be combined to yield

$$(a_{ijC} - a_{ijT}) + \frac{4M^3 \beta^3 S}{\pi^2} \left[ \frac{1}{(i^2 + j^2 M^2 \beta^2)^2 - k_C i^2 M^2 \beta^2} + \frac{1}{R(i^2 + j^2 M^2 \beta^2)^2 + k_T i^2 M^2 \beta^2} \right] \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (a_{mnC} - a_{mnT}) \left( \sum_{c=1}^{M-1} \sin \frac{m\pi c}{M} \sin \frac{i\pi c}{M} \right) \left( \sum_{d=1}^{N-1} \sin \frac{n\pi d}{N} \sin \frac{j\pi d}{N} \right) = 0 \quad (9)$$

$$\begin{aligned} & (i = 1, 2, \dots, \infty) \\ & (j = 1, 2, \dots, \infty) \end{aligned}$$

unless  $a_{ijC} = a_{ijT}$ , in which case

$$\begin{aligned} k_C &= k_T \\ &= \left( \frac{1}{M\beta} + j^2 \frac{M\beta}{1} \right)^2 \end{aligned} \quad \begin{aligned} & (i = 1, 2, \dots, \infty) \\ & (j = 1, 2, \dots, \infty) \end{aligned} \quad (10)$$

which indicates that both plates are subjected to their respective Euler buckling loads.

The remaining details of the analysis are similar to those of references 3 and 6 and are therefore omitted. In summary, the series

$$\left( \sum_{c=1}^{M-1} \sin \frac{m\pi c}{M} \sin \frac{i\pi c}{M} \right) \left( \sum_{d=1}^{N-1} \sin \frac{n\pi d}{M} \sin \frac{j\pi d}{M} \right)$$

may be evaluated by means of the table on page 8 of reference 3. Equations (9) are then seen to separate into independent sets, each of which may be manipulated to yield a stability criterion. The resulting criteria are

$$\begin{aligned} \frac{1}{S} - \frac{\beta^3 N}{\pi^2} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} & \left( \frac{1}{k_C \beta^2 \left( 2s + \frac{q}{M} \right)^2 - \left[ \left( 2s + \frac{q}{M} \right)^2 + (2rN + p)^2 \beta^2 \right]^2} + \right. \\ & \frac{1}{k_C \beta^2 \left[ 2(s+1) - \frac{q}{M} \right]^2 - \left\{ \left[ 2(s+1) - \frac{q}{M} \right]^2 + (2rN + p)^2 \beta^2 \right\}^2} + \\ & \frac{1}{k_C \beta^2 \left( 2s + \frac{q}{M} \right)^2 - \left\{ \left( 2s + \frac{q}{M} \right)^2 + [2(r+1)N - p]^2 \beta^2 \right\}^2} + \\ & \left. \frac{1}{k_C \beta^2 \left[ 2(s+1) - \frac{q}{M} \right]^2 - \left\{ \left[ 2(s+1) - \frac{q}{M} \right]^2 + [2(r+1)N - p]^2 \beta^2 \right\}^2} \right) + \\ \frac{\beta^3 N}{\pi^2 R} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} & \left( \frac{1}{k_T \beta^2 \left[ 2s + \frac{q}{M} \right]^2 + \left[ \left( 2s + \frac{q}{M} \right)^2 + (2rN + p)^2 \beta^2 \right]^2} + \right. \\ & \frac{1}{k_T \beta^2 \left[ 2(s+1) - \frac{q}{M} \right]^2 + \left\{ \left[ 2(s+1) - \frac{q}{M} \right]^2 + (2rN + p)^2 \beta^2 \right\}^2} + \\ & \frac{1}{k_T \beta^2 \left( 2s + \frac{q}{M} \right)^2 + \left\{ \left( 2s + \frac{q}{M} \right)^2 + [2(r+1)N - p]^2 \beta^2 \right\}^2} + \\ & \left. \frac{1}{k_T \beta^2 \left[ 2(s+1) - \frac{q}{M} \right]^2 + \left\{ \left[ 2(s+1) - \frac{q}{M} \right]^2 + [2(r+1)N - p]^2 \beta^2 \right\}^2} \right) = 0 \quad (11) \end{aligned}$$

(q = 1, 2, . . . M-1)  
(p = 1, 2, . . . N-1)

and

$$k_C = -k_T$$

$$= \left( \frac{s}{\beta} + r^2 \frac{M^2 \beta}{s} \right)^2 \quad \begin{matrix} (r = 1, 3, 5, \dots \infty) \\ (s = 1, 3, 5, \dots \infty) \end{matrix} \quad (12)$$

and

$$k_C = -k_T$$

$$= \left( \frac{s}{\beta} + r^2 \frac{M^2 \beta}{s} \right)^2 \quad \begin{matrix} (r = 2, 4, 6, \dots \infty) \\ (s = 2, 4, 6, \dots \infty) \end{matrix} \quad (13)$$

Equations (10) to (13) constitute the complete set of stability criteria for the problem.

#### Computation Details

The computations may be divided into three main groups:

(a) Find the minimum post stiffness required for the compression cover plates to buckle with longitudinal or transverse nodes through the posts.

(b) In those regions where the required stiffness does not exist, find the buckling load attainable with infinitely stiff posts.

(c) Find the boundaries of the various regions.

Equation (11), which was used to make the calculations, involves nine quantities, eight of which must be known in order that the ninth may be determined. These nine quantities are the post axial-stiffness parameter ( $Fb^2/\pi^2 D_C$  or  $S$ ), the ratio of the distance between posts and the box-beam width ( $L/b$  or  $\beta$ ), the two axial-load parameters ( $N_{x_C} b^2/\pi^2 D_C$  or  $k_C$  and  $N_{x_T} b^2/\pi^2 D_T$  or  $k_T$ ), the ratio of the flexural stiffnesses of the tension and compression cover ( $D_T/D_C$  or  $R$ ), the number of bays in the longitudinal and transverse directions ( $M$  and  $N$ ), and the integers which indicate the number of buckles in the longitudinal and transverse directions ( $q$  and  $p$ ). Values of these quantities are determined by the details of the problem to be solved. In all the computations for the present paper, the box beam has been taken to be infinitely long.



Buckling with transverse nodes through the posts.- If the compression cover buckles with transverse nodes through the posts, the buckling-load coefficient is

$$k_C = \left( \frac{1}{\beta} + \beta \right)^2$$

which is the compressive-buckling-load coefficient of a simply supported plate of length  $L$  and width  $b$ . The load coefficient in the tension cover was chosen for the present paper as

$$k_T = Rk_C$$

The minimum required post axial-stiffness parameter is completely determined by equation (11) for specified values of the cover flexural-stiffness ratio, the length-width ratio, and the number of rows of posts, if the number of buckles in the longitudinal and transverse directions are also known.

The ratio  $q/M$  is indicative of the number of buckles in a longitudinal bay of the buckled plate. It seemed reasonable, in view of the results of references 3 to 5, to conclude that  $q/M$  should be taken as unity, since it may be assumed that the transition from buckling with deformation of the posts to buckling with transverse nodes through the posts is continuous for structures with an infinite number of longitudinal bays and not abrupt as would be the case if the number of longitudinal bays were finite. One buckle occurs in the transverse direction and, therefore,  $p$  was taken equal to unity.

Buckling with longitudinal nodes through the posts.- When the ratio of the longitudinal spacing between posts to the box-beam width becomes small, the buckle pattern abruptly changes from one with transverse nodes through the posts to one with longitudinal nodes through the posts. The value of the width ratio at which this phenomenon occurs was found by equating the buckling loads for the two types of buckling because the two are equal at the point of transition. The compressive-buckling-load coefficient for a long simply supported plate of width  $b/N$  is given by

$$k_C = 4N^2$$

The critical length-width ratio was, therefore, found from the relationship

$$4N^2 = \left( \frac{1}{\beta} + \beta \right)^2$$

or

$$\beta_{cr} = N \left( 1 - \sqrt{1 - \frac{1}{N^2}} \right)$$

The minimum post axial-stiffness parameter required for buckling to occur with longitudinal nodes through the posts can be found from equation (11) provided that the ratio  $q/M$  and the number of transverse buckles are known. The calculations of reference 7 for compressive buckling of plates with longitudinal stiffeners indicate that the change from buckling with deflection of the stiffeners to buckling with nodes at the stiffeners is abrupt and that only one transverse buckle should be assumed for the determination of minimum required flexural stiffness. It seemed logical that this condition should be assumed to hold also for plates stiffened by posts and, accordingly,  $p$  was taken equal to unity. The value of  $q/M$  was determined by graphically minimizing the post axial-stiffness parameter with respect to  $q/M$ ; that is, various values of  $q/M$  were chosen, the corresponding values of  $S$  were obtained from equation (11), and the correct value of  $S$  was determined by drawing a curve through the computed values and picking off the minimum.

Boundary between regions.- A third type of buckling is found to appear for relatively low values of the cover flexural-stiffness ratio. For instance, when  $R$  was equal to 2.00 the minimum post axial stiffness required for either longitudinal or transverse node buckle patterns was found to be unobtainable for a certain range of values of the length-width ratio. Further investigation showed that the post axial stiffness required for a longitudinal node buckle pattern became infinite at a certain value of the length-width ratio  $\beta$  less than  $\beta_{cr}$  and that the post axial stiffness required for a transverse node buckle pattern became infinite at some value of  $\beta$  greater than  $\beta_{cr}$ . Between these two values, only buckle patterns with deformation of the posts are obtainable. (If the posts are rigid they displace axially but are not compressed.)

The maximum value of the cover flexural-stiffness ratio  $R$  at which this phenomenon is found to occur was determined by setting the post axial-stiffness parameter equal to infinity, the length-width ratio equal to  $\beta_{cr}$ , the load coefficients  $k_C$  and  $k_T/R$  equal to  $\left(\frac{1}{\beta_{cr}} + \beta_{cr}\right)^2$ , the ratio  $q/M$  and the number of transverse buckles equal to unity, and solving equation (11) graphically. This procedure determines the value of  $R$  for which the spread of the range of  $\beta$ , in which the third type of buckle pattern occurs, vanishes and thus determines the maximum value of  $R$  for which the phenomenon occurs.

The boundaries between the regions in which the various buckle patterns occur were found by placing the post axial stiffness equal to infinity and using a procedure similar to that of the previous sections to determine the length-width ratio  $\beta$  for various values of  $R$ .

When the cover flexural-stiffness ratio is approximately 1.5, the value of  $\beta$  marking the transition from buckling with longitudinal

nodes through the posts and buckling with displacement of the rigid posts is found to become equal to zero, and thus indicates that the longitudinal node buckle pattern is unobtainable for values of  $R$  less than 1.5. Because the stability criterion becomes indeterminate when  $\beta$  is equal to zero, more accurate values of the cover flexural-stiffness ratio for which the phenomenon occurs were found by placing  $\beta$  equal to the low value of 0.01 and finding  $R$  graphically from equation (11).

Buckling loads attainable with rigid posts. - The buckling loads for various values of  $R$  and  $\beta$  in the region where the compression cover buckles with axial displacement of the rigid posts were found by graphically minimizing the loads determined from equation (11) with respect to  $q/M$ , with the post axial-stiffness parameter equal to infinity, and with one buckle in the transverse direction.

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TABLE I  
DATA FOR ONE ROW OF POSTS

R = ∞				R = 2				R = 1			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	2.4	4.340	A	0.75	2.6	4.341	A	0.75	2.7	4.340	A
.50	15.9	6.250	A	.50	20.6	6.250	A	.50	24.4	6.250	A
.45	24.4	7.141	A	.45	34.6	7.141	A	.45	46.5	7.141	A
.40	39.3	8.410	A	.40	63.4	8.410	A	.40	102.8	8.410	A
.37	54.9	9.441	A	.37	101.7	9.441	A	.37	217.5	9.441	A
.35	70.7	10.286	A	.35	151.2	10.286	A	.35	544.4	10.286	A
.33	94.6	11.292	A	.33	261.5	11.292	A	.335	∞	11.0	A or B
.30	163.2	13.201	A	.31	659.3	12.502	A	.25	∞	11.6	B
.28	266.1	14.834	A	.30	1715.4	13.201	A	.10	∞	14.9	B
.268	404.9	16.000	A or C	.294	∞	13.6	A or B				
.225	117.7	16.000	C	.25	∞	15.3	B				
.20	94.9	16.000	C	.207	∞	16.000	B or C				
.15	65.0	16.000	C	.15	522.6	16.000	C				
.10	42.5	16.000	C	.10	253.7	16.000	C				
.05	20.9	16.000	C	.075	176.1	16.000	C				
				.05	118.9	16.000	C				

R = 0.5				R = 0.25			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	2.8	4.340	A	0.75	3.0	4.340	A
.54	22.2	5.721	A	.60	14.9	5.138	A
.50	34.1	6.250	A	.54	32.4	5.721	A
.45	84.3	7.141	A	.50	60.6	6.250	A
.43	137.7	7.593	A	.48	134.4	6.571	A
.41	337.3	8.117	A	.47	222.5	6.748	A
.40	566.4	8.410	A	.46	434.1	6.937	A
.388	∞	8.8	A or B	.449	∞	7.2	A or B
.25	∞	11.6	B	.25	∞	10.0	B
.10	∞	12.5	B	.10	∞	10.6	B
.05	∞	12.5	B	.05	∞	10.6	B

- <sup>a</sup>A Buckling with transverse nodes through the posts.  
 B Buckling with deflection of the posts.  
 C Buckling with longitudinal nodes through the posts.



TABLE II  
DATA FOR TWO ROWS OF POSTS

R = ∞				R = 2				R = 1			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	1.6	4.340	A	0.75	1.7	4.340	A	0.75	1.8	4.340	A
.50	9.8	6.250	A	.50	11.9	6.250	A	.50	13.4	6.250	A
.40	21.1	8.410	A	.40	27.7	8.410	A	.40	33.6	8.410	A
.35	32.3	10.286	A	.35	46.2	10.286	A	.35	59.6	10.286	A
.30	56.7	13.201	A	.30	86.1	13.201	A	.30	125.1	13.201	A
.27	84.9	15.970	A	.27	143.4	15.970	A	.27	248.6	15.970	A
.25	112.9	18.063	A	.25	209.5	18.063	A	.25	450.7	18.063	A
.22	193.8	22.710	A	.22	490.8	22.710	A	.235	1101.4	20.163	A
.20	313.3	27.040	A	.21	767.9	24.720	A	.219	∞	22.9	A or B
.18	598.6	32.897	A	.20	1526.7	27.040	A	.15	∞	28.8	B
.172	888.0	36.000	A or C	.189	∞	30.0	A or B	.10	∞	30.8	B
.15	292.3	36.000	C	.15	∞	34.8	B	.05	∞	30.8	B
.10	157.5	36.000	C	.126	∞	36.000	B or C				
.05	75.2	36.000	C	.10	1933.5	36.000	C				
				.075	918.7	36.000	C				
				.05	597.8	36.000	C				
				.025	282.7	36.000	C				

R = 0.5				R = 0.25			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	1.9	4.340	A	0.75	1.9	4.340	A
.50	15.8	6.250	A	.50	17.9	6.250	A
.40	44.0	8.410	A	.45	35.0	7.141	A
.35	93.6	10.286	A	.40	81.5	8.410	A
.30	330.1	13.201	A	.37	148.8	9.441	A
.28	813.0	14.834	A	.35	289.8	10.286	A
.27	7130.5	15.970	A	.33	1648.6	11.292	A
.268	∞	15.995	A or B	.325	∞	11.6	A or B
.20	∞	20.9	B	.20	∞	17.0	B
.15	∞	22.9	B	.15	∞	18.3	B
.10	∞	24.4	B	.10	∞	19.4	B
.05	∞	24.4	B	.05	∞	19.4	B

- A Buckling with transverse nodes through the posts.  
 B Buckling with deflection of the posts.  
 C Buckling with longitudinal nodes through the posts.



TABLE III  
DATA FOR THREE ROWS OF POSTS

R = ∞				R = 2				R = 1			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	1.2	4.340	A	0.75	1.2	4.340	A	0.75	1.3	4.340	A
.50	7.2	6.250	A	.50	8.6	6.250	A	.50	9.6	6.250	A
.34	26.1	10.766	A	.34	34.7	10.766	A	.34	42.3	10.766	A
.25	70.6	18.063	A	.25	104.1	18.063	A	.29	76.5	13.975	A
.20	153.1	27.040	A	.20	260.8	27.040	A	.25	142.5	18.063	A
.17	279.0	36.331	A	.17	595.2	36.331	A	.20	456.2	27.040	A
.15	527.5	46.467	A	.15	2295.5	46.467	A	.18	1253.7	32.897	A
.14	774.9	53.040	A	.14	∞	53.0	A or B	.17	3473.9	36.331	A
.127	1558.4	64.000	A or C	.125	∞	59.1	B	.164	∞	39.2	A or B
.10	420.4	64.000	C	.11	∞	62.3	B	.10	∞	50.3	B
.08	308.2	64.000	C	.087	∞	64.000	B or C	.05	∞	52.8	B
.04	144.1	64.000	C	.03	1151.2	64.000	C				
				.02	752.3	64.000	C				
				.01	357.3	64.000	C				

R = 0.5				R = 0.25			
β	S	k	Type of buckling (a)	β	S	k	Type of buckling (a)
0.75	1.3	4.340	A	0.75	1.4	4.340	A
.50	10.9	6.250	A	.50	12.7	6.250	A
.34	56.5	10.766	A	.40	36.5	8.410	A
.32	69.2	11.868	A	.34	87.5	10.766	A
.29	114.8	13.975	A	.32	122.3	11.868	A
.27	167.7	15.970	A	.29	247.4	13.975	A
.25	263.3	18.063	A	.27	467.2	15.970	A
.23	534.8	20.957	A	.26	1781.9	16.860	A
.21	2097.5	24.720	A	.254	∞	17.6	A or B
.205	∞	25.8	A or B	.175	∞	25.3	B
.10	∞	38.9	B	.10	∞	29.9	B
.05	∞	40.3	B	.05	∞	30.8	B

- <sup>a</sup>  
A Buckling with transverse nodes through the posts.  
B Buckling with deflection of the posts.  
C Buckling with longitudinal nodes through the posts.



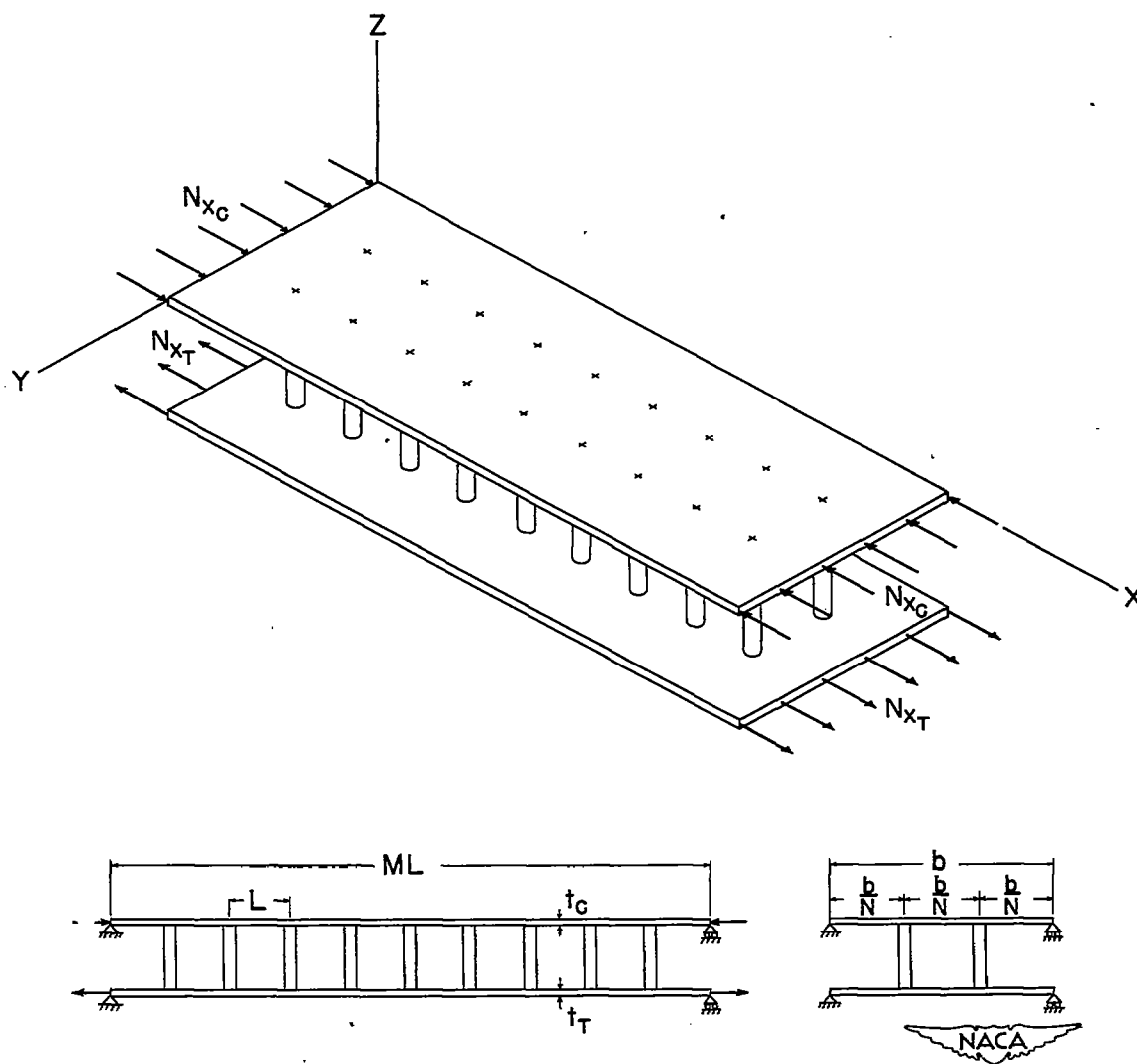
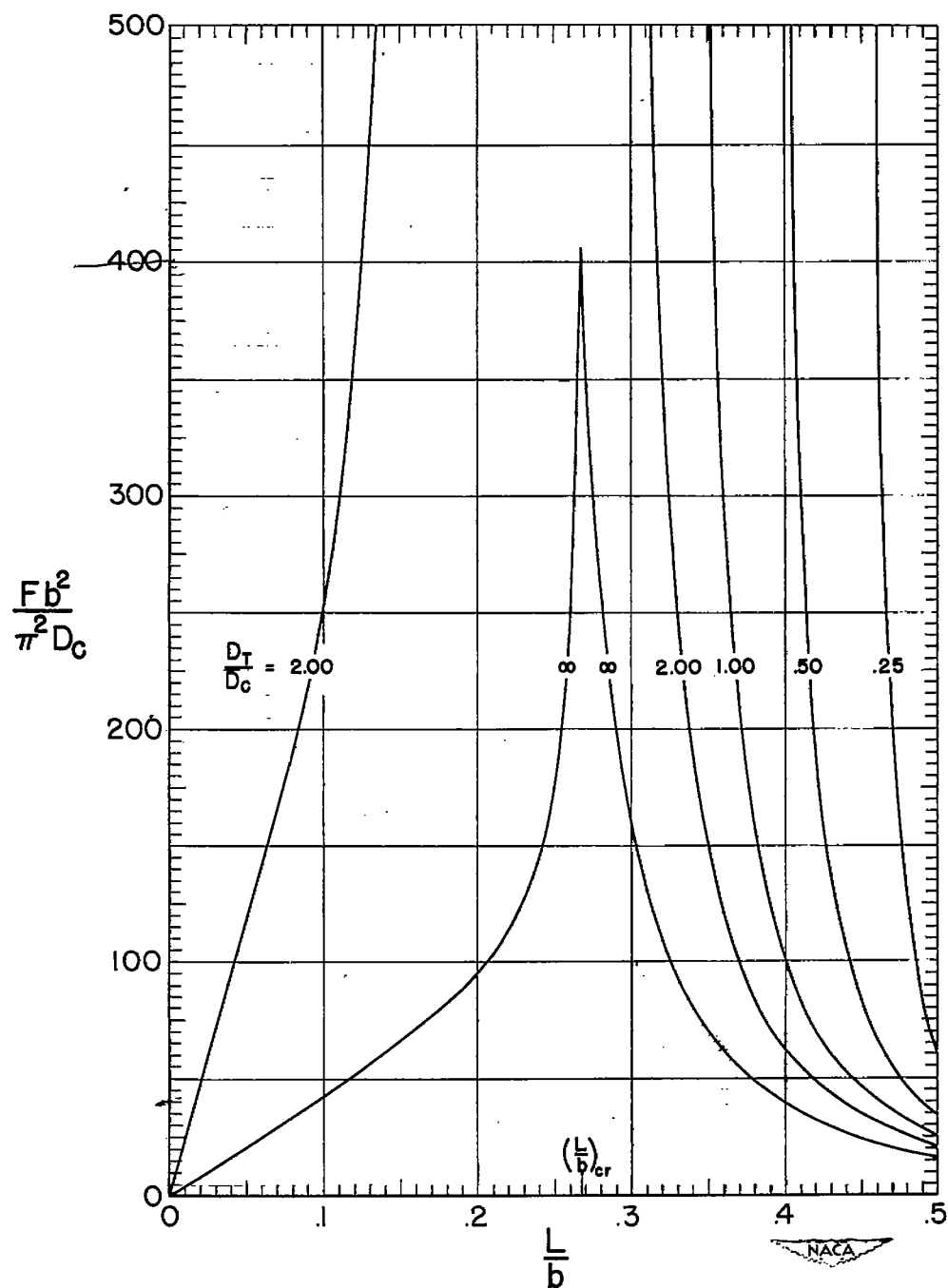


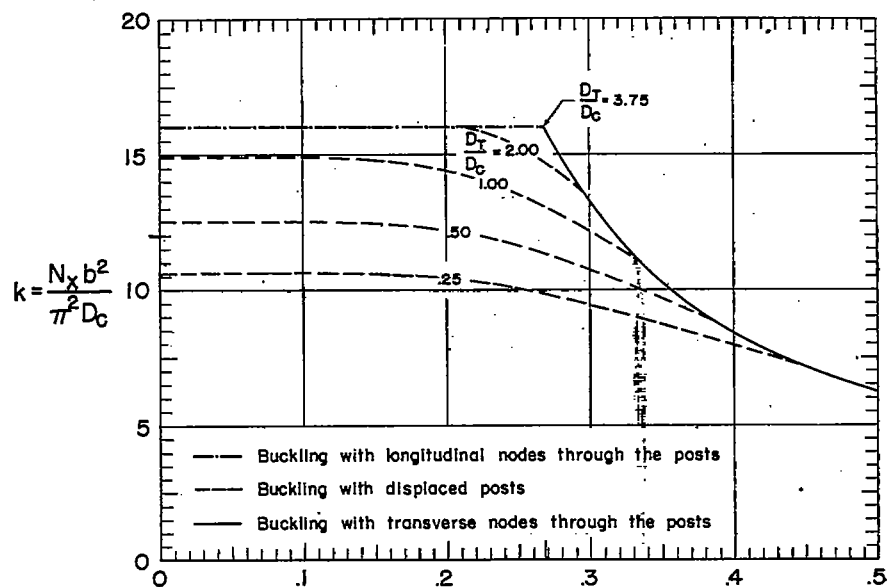
Figure 1.- Idealization of box beam stiffened by posts.



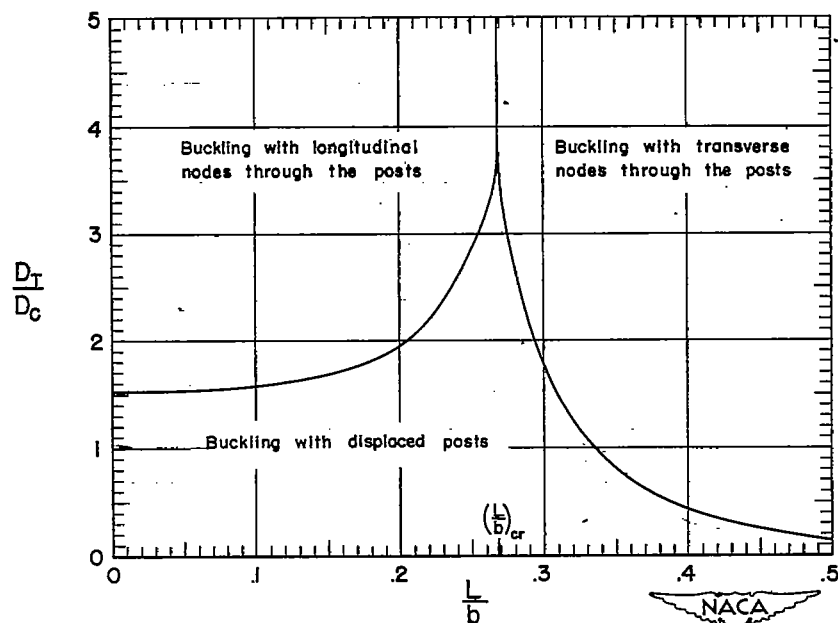


(a) Minimum post axial stiffness required for compression cover to buckle with nodes through the posts.

Figure 2.- One row of posts.

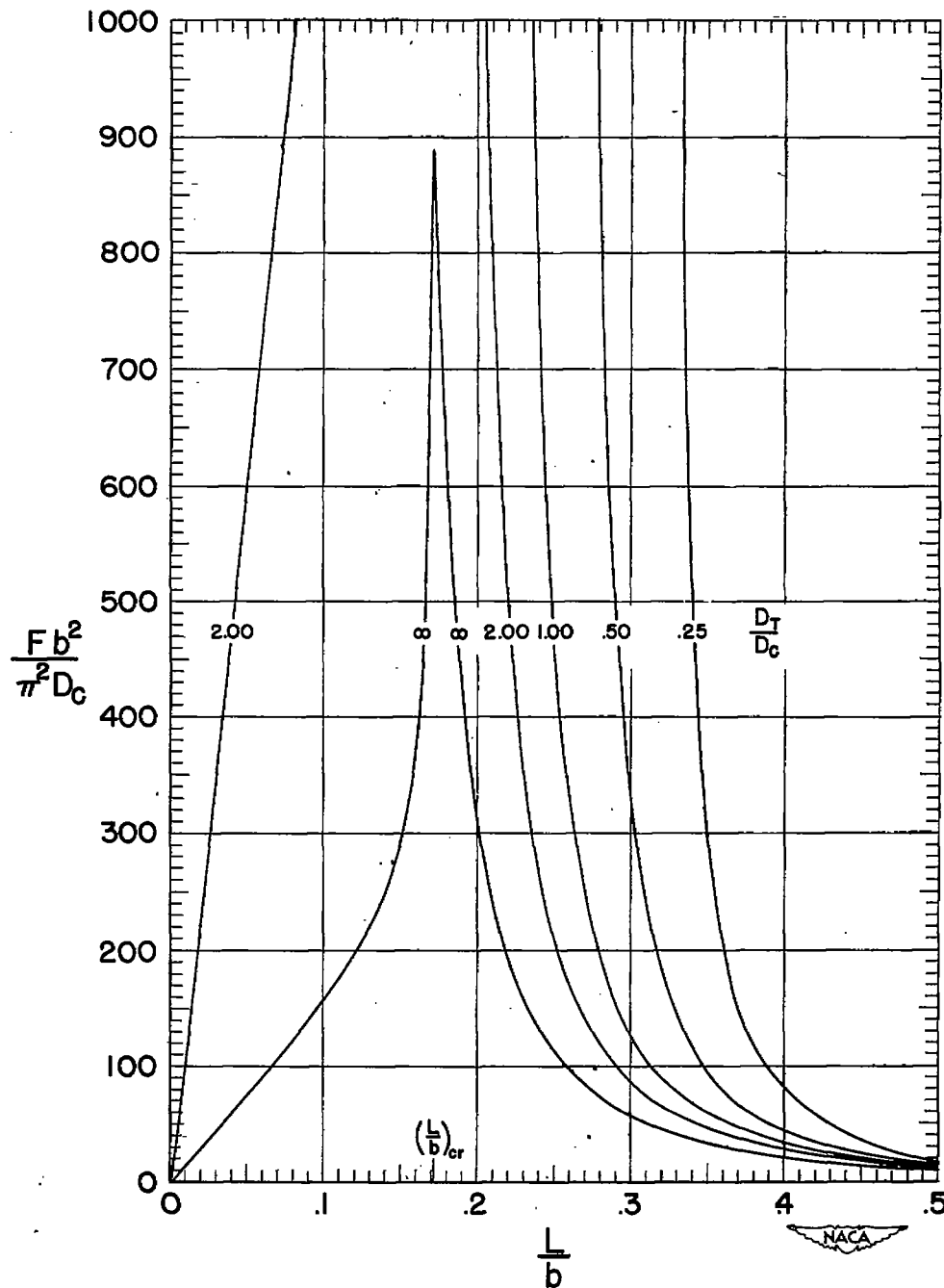


(b) Buckling loads for box beams with effectively rigid posts.



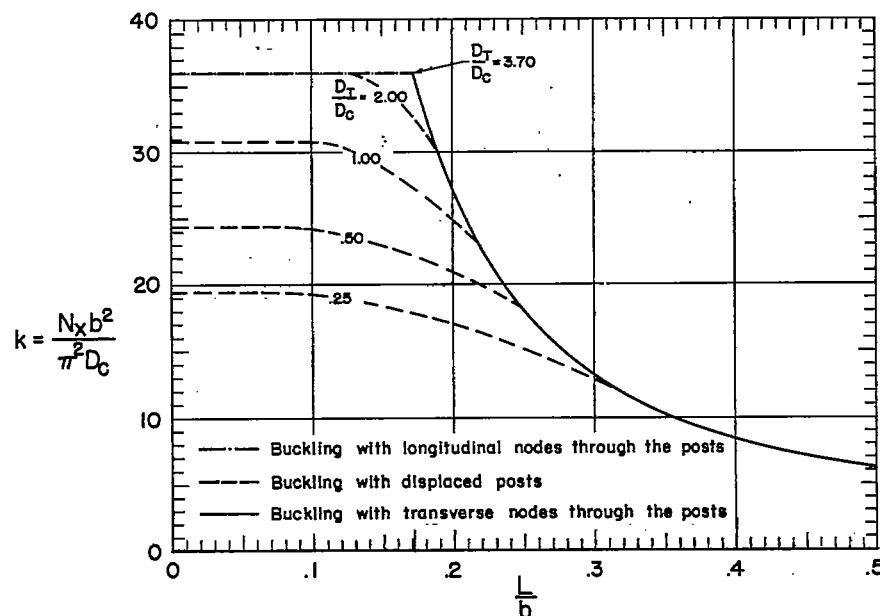
(c) Buckling phenomena attainable for box beams with effectively rigid posts.

Figure 2.- Concluded.

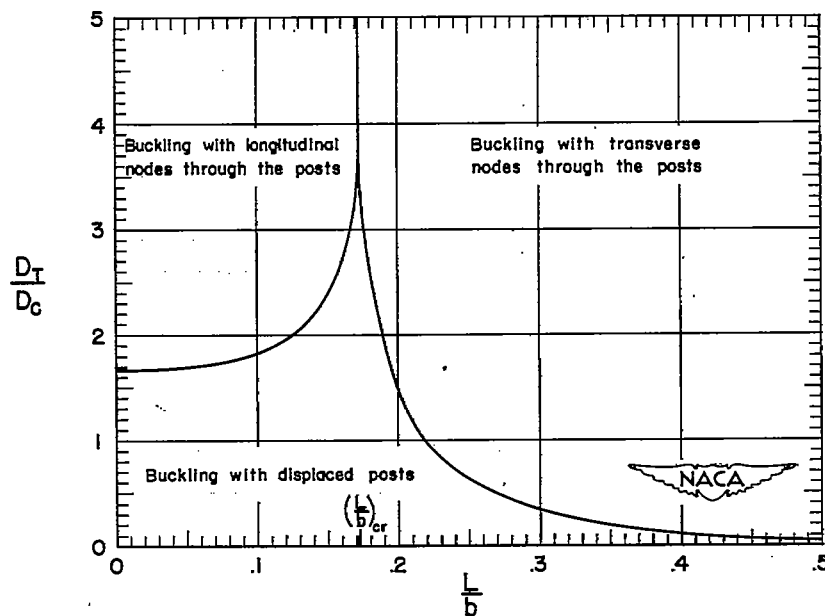


(a) Minimum post axial stiffness required for compression cover to buckle with nodes through the posts.

Figure 3.- Two rows of posts.

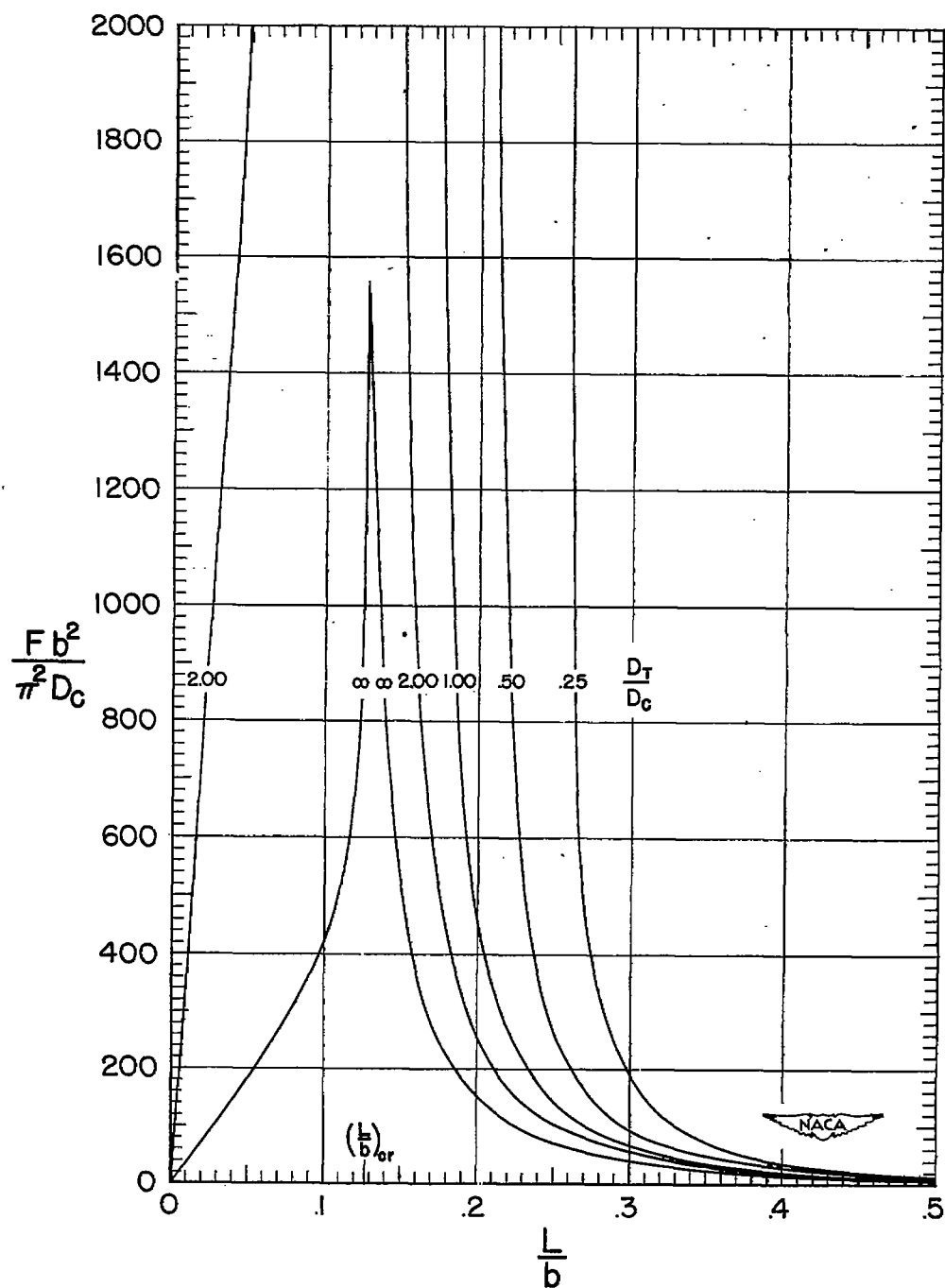


(b) Buckling loads for box beams with effectively rigid posts.



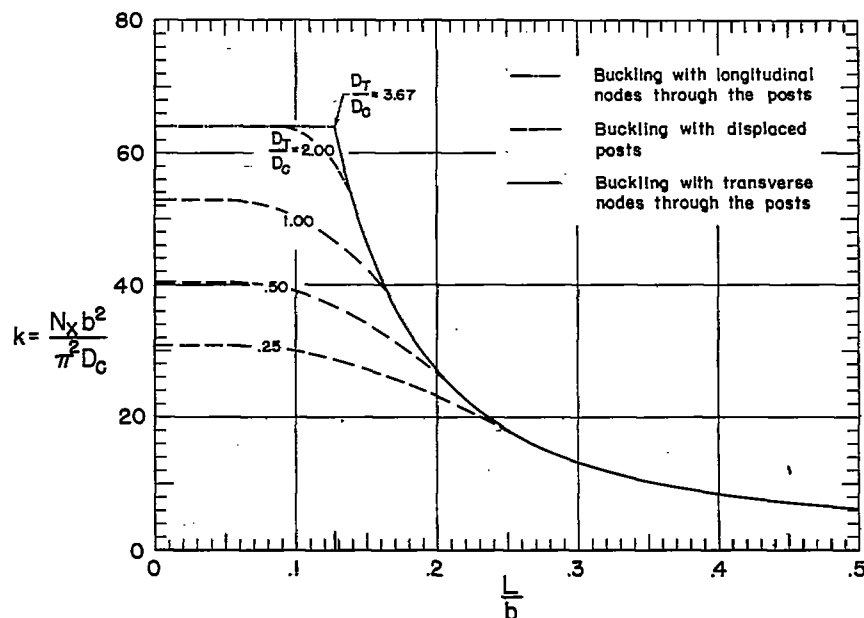
(c) Buckling phenomena attainable for box beams with effectively rigid posts.

Figure 3.- Concluded.

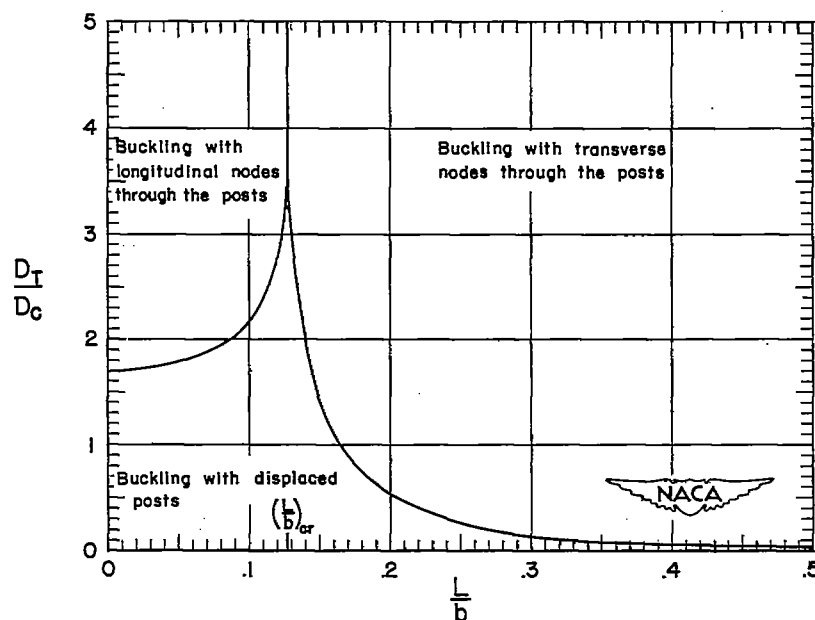


(a) Minimum post axial stiffness required for compression cover to buckle with nodes through the posts.

Figure 4.- Three rows of posts.



(b) Buckling loads for box beams with effectively rigid posts.



(c) Buckling phenomena attainable for box beams with effectively rigid posts.

Figure 4.- Concluded.